Composition, translation and stretching, the inverse function

Submission instructions: Clearly write your full name and ID number on the first page. To avoid marking and handling difficulties, please staple all submitted pages together and answer the questions in the order they appear on the assignment.

Academic integrity: Students are encouraged to collaborate on assignment problems but must write up their assignments independently. Copying is strictly forbidden!

Suggested problems:

Section 1.4, 1 – 6
Section 1.8, 10 – 24, 25 – 36, 41 – 54

Problems for submission:

1. Consider the functions \( f(x) = \frac{1}{x+1} \) and \( g(x) = \frac{1}{x-1} \).

   (a) Find the domain and range of \( f(x) \) and \( g(x) \).
   (b) Find \( f \circ g \).
   (c) What is the domain of \( f \circ g \)? \([\textbf{Hint:} \text{Do not forget to consider the domain of } g(x)!]\)

2. Find the inverses \( f^{-1}(x) \) of the following functions

   (a) \( f(x) = \frac{2x - 3}{7 - x} \), \hspace{1cm} (b) \( f(x) = 2 - \ln(x^3 - 1) \).

   (c) \( f(x) = \frac{p(x) - 1}{2 + 3p(x)} \) (assume \( p(x) \) is invertible)
(d) \( f(x) = \begin{cases} 
\sqrt{-x-1}, & \text{for } x \leq -2 \\
-x, & \text{for } -2 < x \leq 0 \\
-\frac{x}{2}, & \text{for } x > 0.
\end{cases} \)

3. Graph the following functions and explain the steps (translation, reflection, etc.) required to go from a basic function to \( f(x) \):

(a) \( f(x) = e^{-x} + 1 \),
(b) \( f(x) = \sin\left(\frac{\pi}{2}x + \pi\right) \),
(c) \( f(x) = -\ln(-x - 1) - 1 \).

4. Simplify the following expressions:

(a) \( \tan(\arcsin(x)) \),
(b) \( \sin\left(\arcsec\left(\frac{x}{5}\right)\right) \),
(c) \( \cos(\arctan(-1)) \).

5. Plot \( f(x) = \arcsin(\sin(x)) \). What is the domain and range?

6. Express the function \( f(x) = -7\sin(x) - 4\cos(x) \) as a single phase-shifted sine function.

7. The number of cells in a culture of bacteria is given as a function of time by the formula

\[
P(t) = \frac{K P_0 e^{rt}}{K + P_0 (e^{rt} - 1)}
\]

where \( K \) is the carrying capacity of the environment, \( r \) is a scale constant, and \( P_0 \) is the initial population size. Find the inverse of this function \( P^{-1}(t) \). Briefly explain what this relationship means.