Partial fraction decomposition, heaviside function, odd and even functions

Submission instructions: Clearly write your full name and ID number on the first page. To avoid marking and handling difficulties, please staple all submitted pages together and answer the questions in the order they appear on the assignment.

Academic integrity: Students are encouraged to collaborate on assignment problems but must write up their assignments independently. Copying is strictly forbidden!

Suggested problems:

Section 6.5, 1 – 12, 21 – 36

(perform partial fraction decomposition only)

Other homework problems to be posted online

Problems for submission:

1. Find the partial fraction decomposition of the following:

   (a) \( \frac{1}{x^2 - 1} \), (b) \( \frac{2x - 1}{x^3 - 2x^2 - x + 2} \), (c) \( \frac{x^3}{x^3 - x^2 + x - 1} \)

   (Hint: It is easy to check whether \( x = 1 \) is a factor of a polynomial. All you have to do is check whether the coefficients add up to zero!)

2. Set up (but do not attempt to evaluate) the partial fraction decomposition of the following:

   \( \frac{x}{x^2(x - 1)(x^2 + x + 3)(x^2 + 1)^3} \).

3. Rewrite the following function as a piecewise-defined function and then graph it:

   \( f(x) = \sin(x) + (\cos(x) - \sin(x))H\left( x + \frac{3\pi}{4} \right) + (\sin(x) - \cos(x))H\left( x - \frac{\pi}{4} \right) \).
4. Rewrite the following piecewise function using the heaviside function:

\[ f(x) = \begin{cases} 
  e^x, & \text{for } x < 0 \\
  -(x - 1)^2 + 2, & \text{for } 0 \leq x < 2 \\
  \cos(\pi x), & \text{for } x \geq 2.
\]

5. Classify the following functions as odd, even, or neither:

(a) \( f(x) = 5x^3 - x \),  
(b) \( f(x) = x^4 - x^2 + 7 \)

(c) \( f(x) = x^4 + 5x^3 - x^2 - x + 7 \)

(d) \( f(x) = 1 - \sin^2(x) \),  
(e) \( f(x) = \ln(|x|) \).

6. Consider the function defined on \( 0 \leq x \leq \pi \) given by the following.

Graph a function defined on \( -\pi \leq x \leq \pi \) which agrees with the plotted function on \( 0 \leq x \leq \pi \) and which is (a) even, and (b) odd.

7. Carbon-14 is an unstable isotope of carbon which is used to date the age of biological samples. The decay of the isotope after death can be modelled by the exponential curve:

\[ A(t) = A_0 e^{kt} \]

where \( A_0 \) is the initial amount of the substance and \( k \) is the decay constant.

(a) The half-life of Carbon-14 is known to be 5730 years. Use this to determine the value of the decay constant \( k \).

(b) If we have an initial sample of 1000 mg, how much of the sample will remain after 2000 years?