1. Use linear approximation to estimate the values of
   \( \sqrt{10} \) \( \ln(1.1) \) \( \sin \left( \frac{\pi}{4} + 0.01 \right) \)

2. Use Newton’s method to estimate the root \( x^* \) of \( f(x) = -x^3 - 2x^2 + 1 \)
   in the interval \( I = [0, 1] \) to five decimal places of accuracy.

3. Use Newton’s method to estimate the value of \( e \) to three decimal places
   of accuracy. [\textbf{Hint:} Consider taking the natural log of \( x = e \).]

4. Find a point \( c \in I \) within the given interval \( I = [a, b] \) for which the
   instantaneous rate of change at \( c \), \( f'(c) \), is equal to the average rate of
   change over the interval, \( \frac{f(b) - f(a)}{b - a} \).
   \( a) f(x) = 2\sqrt{x} \) over the interval \( I = [1, 4], \)
(b) \( f(x) = \frac{1}{1 + x} \) over the interval \( I = [0, 1] \), and

(c) \( f(x) = \sqrt{4 - x^2} \) over the interval \( I = [0, 2] \).

5. Show that if \( f(x) \) is a parabola defined over the interval \( I = [a, b] \) then the point \( c \in (a, b) \) guaranteed by the Mean Value Theorem is always the midpoint, i.e. \( c = \frac{a + b}{2} \).

6. Use L’Hôpital’s rule (if applicable) to evaluate the following limits:

(a) \( \lim_{x \to 0} x[\ln(x)]^2 \)

(b) \( \lim_{x \to \infty} \frac{\sqrt{x^2 - 1}}{x} \)

(c) \( \lim_{t \to 0} \frac{\sin(kt)}{t} \), \( k \neq 0 \)

(d) \( \lim_{x \to \infty} \left( 1 + \frac{1}{x} \right)^x \).

7. L’Hôpital’s rule can be used to determine which classes of functions grow faster than others. That to say, it can resolve the question of whether \( f(x) > g(x) \) for large \( x \) or \( g(x) > f(x) \) for large \( x \). Consider the functional classes of polynomials \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \) and exponentials \( g(x) = b_n x^n + b_{n-1} x^{n-1} + \cdots + b_0 \) where \( a_0, \ldots, a_n, b_0, \ldots, b_n, \) and \( n \) are fixed values. Which class of functions grows faster, polynomials or exponentials?

[**Hint:** Consider what happens in the limit \( \lim_{x \to \infty} \frac{f(x)}{g(x)} \).]