Related rates, local and global extrema, optimization

**Submission instructions:** Clearly write your full name and ID number on the first page. To avoid marking and handling difficulties, please staple all submitted pages together and answer the questions in the order they appear on the assignment.

**Academic integrity:** Students are encouraged to collaborate on assignment problems but must write up their assignments independently. **Copying is strictly forbidden!**

**Suggested problems:**

Section 4.2, 1 – 26
Section 4.4, 1 – 8, 33 – 34
Section 4.7, 1 – 2, 7 – 28

**Problems for submission:**

1. Find and classify all extrema of the following functions:
   
   (a) \( f(x) = x^4 - 4x^3 - 2x^2 + 12x + 1, \quad 0 \leq x \leq 4 \)
   
   (b) \( f(x) = \sin^3(x), \quad -\pi/2 \leq x \leq \pi/2 \)
   
   (c) \( f(x) = \begin{cases} 
   4 - (x - 1)^2, & \text{for } 0 \leq x < 3 \\
   x - 3, & \text{for } 3 \leq x < 6. 
   \end{cases} \)

2. A 10cm metal wire is to be divided into two pieces. The first half of the wire is to be folded into a circle and the second half is going to be folded into a square. Determine where to make the cut if we are interested in minimizing the combined area of the circle and the square.

3. Determine the point on the curve \( f(x) = \sqrt{x} \) closest to the point \((3/2, 0)\).
4. Two trains leave the train station at the same time. The first train travels due east at 120 km/hr and the second travels due north at 90 km/hr. How quickly are the trains moving apart when they have been travelling for 40 minutes?

5. (BONUS) Consider a particle moving counter-clockwise around the circle $x^2 + y^2 = r^2$. Let $\theta$ denote the angle in radians relative to the positive $x$-axis. Suppose that $\frac{d\theta}{dt} = 1$. Determine $\frac{dx}{dt}$ and $\frac{dy}{dt}$ when $\theta = \frac{3\pi}{4}$. (Hint: Use trigonometric identities to relate $x$ and $y$ to $\theta$.)