Submission instructions: Clearly write your full name and ID number on the first page. To avoid marking and handling difficulties, please staple all submitted pages together and answer the questions in the order they appear on the assignment.

Academic integrity: Students are encouraged to collaborate on assignment problems but must write up their assignments independently. Copying is strictly forbidden!

Suggested problems:

Section 4.3, 1 – 24 (odd), 37 – 42, 45 – 52
Section 4.6, 1 – 22 (odd)

Problems for submission:

1. Determine the second derivatives $\frac{d^2y}{dx^2}$ for the following functions:
   
   (a) $y = e^{-x} \sin(x)$
   (b) $y = \arctan(x)$
   (c) $x^2 + y^2 = 1$

   (Hint: We will have to implicitly differentiate twice then use our solution for $\frac{dy}{dx}$ and the original equation $x^2 + y^2 = 1$ to solve for $\frac{d^2y}{dx^2}$ in terms of $x$ and $y$.)

2. Use the second derivative test to determine which of the critical points of the following functions correspond to local maximums and which correspond to local minimums.
   
   (a) $f(x) = x^3 - 3x - 1$
(b) \( f(x) = \sin(x) \)
(c) \( f(x) = x \ln(x) \)

3. Use information about \( f(x) \), \( f'(x) \), and \( f''(x) \) to sketch the following graphs:

(a) \( f(x) = \frac{x^2 + x - 1}{x + 2} \)
(b) \( f(x) = \ln\left(\frac{x - 1}{x + 1}\right) - x \)

(Hint: Be careful with the domain! You may assume the function has no roots and is positive in the region \( x < -1 \) and negative in the region \( x > 1 \).)

**BONUS:** The linear approximation formula \( L(x) = f(a) + f'(a)(x - a) \) is well chosen to approximate \( f(x) \) in a neighbourhood of \( x = a \) because it agrees with \( f(x) \) and its first derivative \( f'(x) \) at \( x = a \) (i.e. \( L(a) = f(a) \) and \( L'(a) = f'(a) \)). We might wonder if higher-order derivatives can give more information. Consider the following questions.

(a) Use linear approximation to estimate \( \sqrt{\frac{17}{25}} \).
(b) Find an extension of \( L(x) \) which agrees with the second derivative of \( f(x) \) as well (i.e. \( L''(a) = f''(a) \)). (Hint: Consider adding a term of the form \( A(x - a)^2 \) and solve for \( A! \))

(c) Use this new function to approximate \( \sqrt{\frac{17}{25}} \). Compare with your calculator value. Is this estimate better than linear approximation? Worse? What do you think would happen if we added terms which made our approximation agree to even higher-order derivatives?