
Topics:

- Taylor series
- Taylor polynomials and remainders
- Binomial series

You are to provide full solutions to the following problems. You are allowed, and encouraged to collaborate with your classmates, use your notes and textbook and ask the TA for guidance. Direct copying of solutions is not encouraged, nor is it allowed or ethical.

Last name: __________________________  First name: __________________________

Student number: __________________________
1. Determine the Taylor series expansion of \( f(x) = x^4 - 2x^2 + x - 1 \) about \( x = -1 \).

2. Determine the Maclaurin series of \( f(x) = \frac{1}{(x + 2)(3x - 1)} \) using partial fractions.

3. Determine the Taylor series expansion of \( f(x) = x \ln(x) - x \) about \( x = 1 \). (Hint: Consider \( f'(x) \).)

4. Determine the Maclaurin series of \( f(x) = \frac{x^2}{(1 + x^2)^3} \) using the binomial expansion.

5. Show that the following identities are valid to the first three non-zero terms of the Maclaurin expansion:
   
   (a) \( \sin(2x) = 2 \sin(x) \cos(x) \)  
   (b) \( \cos(2x) = 2 \cos^2(x) - 1 \)

6. (a) Determine the Taylor series expansion of \( \ln(x) \) about \( x = a \) where \( a > 0 \).
   (b) Determine the interval of convergence of your expansion (including endpoints).
   (c) This expansion is of limited computational use since we can only evaluate \( \ln(a) \) explicitly for \( a = 1 \). However, we know that

   \[
   \ln(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (x - 1)^n
   \]

   converges on \( 0 < x \leq 2 \). Derive a series which converges to \( \ln(2) \).
   (d) Use part (c) to derive a single series (i.e. single sum) which converges to \( \ln(x) \) on \( 0 < x \leq 4 \).
   (e) **(Bonus)** Derive a single series which converges to \( \ln(x) \) on \( 0 < x \leq 8 \).