MATH 320, Spring 2013, Assignment 7
Due date: Friday, March 22

Name (printed): ________________________________

UW Student ID Number: ___________________________

Discussion Section: (circle)
Robin Prakash: 301 302 303
Sowmya Acharya: 304 306 307 308
Raghvendra Chaubey: 352 353 354 355

Instructions

1. Fill out this cover page completely and affix it to the front of your submitted assignment.

2. Staple your assignment together and answer the questions in the order they appear on the assignment sheet.

3. Show all the work required to obtain your answers.

4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. Copying is strictly forbidden!

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Determinants, Vector Spaces

Suggested problems:

Section 3.6: 1-6, 13-53, 59, 60
Section 4.1: 1-41

Problems for submission:

Section 3.6: 6, 16, 21, 30, 37, 52
Section 4.1: 16, 17, 20

(Justify your answers for full marks!)

1. A matrix parameter which often arises in conjunction with the determinant is the trace. The trace of an \( n \times n \) matrix \( A \) is defined as

\[
\text{tr}(A) = \sum_{i=1}^{n} a_{ii}.
\]

In other words, it is the sum of the elements along the diagonal. To see one application where the trace arises, consider the following problem (which we will reconsider in more depth in a few weeks).

Consider an arbitrary \( 2 \times 2 \) matrix \( A = [a_{ij}] \). Suppose we are interested in manipulating \( A \) by subtracting (or adding) a fixed value from the diagonal (i.e. we are interested in the matrix \( B = A - \lambda I \) where \( \lambda \in \mathbb{R} \)).

(a) Show that the condition \( \det(B) = 0 \) gives the characteristic polynomial

\[
\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0. \tag{1}
\]

(The solutions \( \lambda \) of the characteristic equation give the possible values of \( \lambda \) for which \( \det(B) = \det(A - \lambda I) = 0 \).)

(b) Show that a matrix \( B \) permits real values \( \lambda \in \mathbb{R} \) satisfying \( \det(B) = 0 \) if and only if \( \text{tr}(A)^2 \geq 4\det(A) \).

(c) Determine the values of \( \lambda \) for which \( \det(B) = \det(A - \lambda I) = 0 \) for the following matrices \( A \):

(i) \( A = \begin{bmatrix} -2 & 1 \\ 0 & 2 \end{bmatrix} \),
(ii) \( A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \),
(iii) \( A = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix} \).

[Hint: This can be done directly for each matrix, but it may save some time to solve for \( \lambda \) in (1) in terms of \( \text{tr}(A) \) and \( \det(A) \).]