Section 5.1 Verify that the functions $y_1(x)$ and $y_2(x)$ are solutions of the given differential equation. Then find a particular solution of the form $y(x) = C_1y_1(x) + C_2y_2(x)$ that satisfies the given initial conditions. Primes denote derivatives with respect to $x$.

# 15. $x^2y'' - xy' + y = 0; y_1 = x, y_2 = x \ln x; y(1) = 7, y'(1) = 2$

Determine whether the following pair of functions are linearly independent or linearly dependent on the real line.

# 20. $f(x) = \pi, g(x) = \cos^2(x) + \sin^2(x)$

# 25. $f(x) = e^x \sin x, g(x) = e^x \cos x$

Section 5.2 Show that the given set of functions are linearly dependent on the real number line. That is, find a non-trivial linear combination of the given functions that vanished identically.

# 6. $f(x) = e^x, g(x) = \cosh(x), h(x) = \sinh(x)$

Use the Wronskian to prove that the given functions are linearly independent on the indicated interval.

# 11. $f(x) = x, g(x) = xe^x, h(x) = x^2e^x; \text{ the real line}$

Section 5.3 Find the general solutions of the following differential equation.

# 7. $4y''' - 12y'' + 9y = 0$

Solve the following initial value problem.

# 23. $y''' - 6y'' + 25y = 0; y(0) = 3, y'(0) = 1$

# 38. Solve the initial value problem

$$y''' - 5y'' + 100y' - 500y = 0;$$
$$y(0) = 0, \quad y'(0) = 10, \quad y''(0) = 250$$

given that $y_1(x) = e^{5x}$ is a particular solution of the differential equation.