Consider two chemical reaction networks \( \mathcal{N} \) and \( \mathcal{N}' \) endowed with mass-action kinetics and let \( \Psi(x, t) \) denote the flow associated with \( \mathcal{N} \) and \( \Psi'(y, t) \) denote the flow associated with \( \mathcal{N}' \). We will say \( \mathcal{N} \) and \( \mathcal{N}' \) are linearly conjugate if there exists a linear mapping \( h : \mathbb{R}^n \rightarrow \mathbb{R}^m \) such that
\[
\Psi'(h(x), t) = \Psi(h(x), t) \quad \text{for all } x \in \mathbb{R}^n.
\]
Linearly conjugate reaction networks are important because they share many of the same qualitative dynamics even if their reaction graphs are different.

### Example

#### Deficiency Three

Consider a network of binary interactions between singulary-bound enzymes with three binding sites:

This network has a deficiency of three and can exhibit a variety of dynamical behaviours, including multiple positive equilibrium states.

#### Deficiency Two

Now choose the rate constants flowing into each complex to be the same. The original network is then linearly conjugate to the following:

This network has a deficiency of two and can be found very quickly using the introduced MILP algorithm.

#### Deficiency One

If we choose the rate constants flowing from each complex to be the same, the original network is linearly conjugate to the following:

This network has a deficiency of one and it can be shown by deficiency one algorithms to exhibit at most one positive equilibrium state.

### References