Antidifferentiation, Riemann sums, indefinite and definite integral

Submission instructions: Clearly write your full name and ID number on the first page. To avoid marking and handling difficulties, please staple all submitted pages together and answer the questions in the order they appear on the assignment.

Academic integrity: Students are encouraged to collaborate on assignment problems but must write up their assignments independently. Copying is strictly forbidden!

Suggested problems:

Section 2.10, 1 – 26
Section 5.2, 1 – 19
Section 5.3, 1 – 10

Problems for submission:

1. Find an antiderivative for the following functions (you can omit arbitrary constants):
   
   (a) $x^{2012}$,  
   (b) $\frac{2}{1-x}$,  
   (c) $\frac{1}{4+x^2}$,  
   (d) $e^x \cos(e^x)$,  
   (e) $\frac{2x + 1}{x^2 + x + 1}$.

2. Truncated Riemann sums can be used to approximate areas even when the limit as $n \to \infty$ cannot be easily evaluated.

   (a) Use the first four Riemann rectangles to approximate the area under the curve $f(x) = e^x$ bound between $x = 0$ and $x = 1$. Use both the left and right endpoints. Can we bound the actual area under the curve based on this information? [Hint: Consider the graph!]
(b) Use the first four Riemann rectangles to approximate the area under the curve \( f(x) = -x \sin(x) \) bound between \( x = \pi \) and \( x = 2\pi \). Use both the left and right endpoints. Can we bound the actual area under the curve based on this information?

3. Verify that the Riemann sum for \( f(x) = -x^2 + 2x \) evaluated between \( x = 0 \) and \( x = 2 \) is the same using the left and right endpoint of each rectangle to determine the height.

4. The choice of point \( x^*_i \) in the Riemann sum formula can be chosen arbitrarily so long as it lies on the base of the rectangle. Another popular choice is the midpoint, i.e.

\[
x^*_i = a + (i - 1/2)\Delta x.
\]

Set up the Riemann sum for \( f(x) = x \ln(x) \) evaluated from \( x = 1 \) to \( x = 5 \) using the midpoint \( x^*_i \). (Do not attempt to evaluate the sum!)

5. Give the definite integral which corresponds to the following terms:

   (a) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n} \ln \left( 1 + \frac{2i}{n} \right) \)

   (b) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{n}{n^2 + i^2} \)

6. Jack has been on the road for half an hour, but his odometer is broken so he has no idea how far he has travelled. He has, however, been conscientious enough to look at his speedometer several times over the period and has jotted down the following values:

<table>
<thead>
<tr>
<th>Table 1: Velocity versus time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (minutes)</td>
</tr>
<tr>
<td>Velocity (km/h)</td>
</tr>
</tbody>
</table>

Based on what you know about the Riemann sum as an estimate of the area under a curve, find an estimate of how far Jack has travelled.