1. Evaluate the following definite integrals:

(a) \( \int_{-1}^{1} \frac{1}{x + 2} \, dx \)

**Solution:**
\[
\int_{-1}^{1} \frac{1}{x + 2} = [\ln(x + 2)]_{-1}^{1} = \ln(3) - \ln(1) = \ln(3).
\]

(b) \( \int_{0}^{\pi/4} -2 \sec^2(x) \, dx \)

**Solution:**
\[
\int_{0}^{\pi/4} -2 \sec^2(x) \, dx = -2 [\tan(x)]_{0}^{\pi/4} = -2 [\tan\left(\frac{\pi}{4}\right) - \tan(0)]
\]

(c) \( \int_{0}^{x} \frac{1}{\sqrt{4 - t^2}} \, dt \)

**Solution:**
\[
\int_{0}^{x} \frac{1}{\sqrt{4 - t^2}} \, dt = \int_{0}^{x} \frac{1}{\sqrt{4(1 - (t/2)^2)}} \, dt = \frac{1}{2} \int_{0}^{x} \frac{1}{\sqrt{1 - (t/2)^2}} \, dt
= \frac{1}{2} \left[2 \arcsin\left(\frac{t}{2}\right)\right]_{0}^{x} = \arcsin\left(\frac{x}{2}\right).
\]

2. Use Liebniz’ rule to evaluate the following:

(a) \( \frac{d}{dx} \int_{0.01}^{x} \ln(\sin(s)) \, ds \), \( 0.01 \leq x < \pi \)
Solution:
\[ \frac{d}{dx} \int_{0.01}^{x} \ln(\sin(s)) \, ds = \ln(\sin(x)). \]

for \( 0.01 \leq x < \pi \). Note that \( \ln(\sin(x)) \) is well-defined in this region.

(b) \( \frac{d}{dx} \int_{\ln(x)}^{x} e^{-t^2} \, dt \)

Solution:
\[ \frac{d}{dx} \int_{\ln(x)}^{x} e^{-t^2} \, dt = e^{-x^2} - e^{-\ln(x)^2} \left( \frac{1}{x} \right) = e^{-x^2} - \frac{e^{-\ln(x)^2}}{x}. \]

(c) \( \frac{d}{dx} \int_{-x}^{0} \cos(\theta) \sin(\theta) \, d\theta \)

Solution:
\[ \frac{d}{dx} \int_{-x}^{0} \cos(\theta) \sin(\theta) \, d\theta = -\cos(-x) \sin(-x)(-1) = -\cos(x) \sin(x). \]

3. Evaluate the following integrals:

(a) \( \int_{-r}^{r} \sqrt{r^2 - x^2} \, dx \)

Solution: The relation \( y = \sqrt{r^2 - x^2} \) can be rearranged into \( x^2 + y^2 = r^2 \). This is the equation of a circle of radius \( r \). The area corresponding to the given integral is the upper semi-circle (half of the full area of a circle). We therefore have
\[ \int_{-r}^{r} \sqrt{r^2 - x^2} \, dx = \frac{1}{2} \pi r^2. \]

(b) \( \int_{0}^{5} \frac{x^2 - 5x + 6}{|x - 2|} \, dx \)

Solution: We have
\[ |x - 2| = \begin{cases} 
x - 2, & \text{for } x \geq 2 \\
-x + 2, & \text{for } x < 2.
\end{cases} \]
It follows that

\[
\int_{0}^{5} \frac{x^2 - 5x + 6}{|x - 2|} dx = \int_{0}^{5} \frac{(x - 2)(x - 3)}{|x - 2|} dx
\]

\[
= \int_{0}^{2} (-x + 3) \, dx + \int_{2}^{5} (x - 3) \, dx.
\]

We can evaluate this geometrically by considering the picture given in Figure 1. We have to be a little careful since one triangle lies below the \(x\)-axis and therefore must have its area subtracted from the others. In total, we have that the area is 5.5 units.

![Figure 1: Area bound between the x-axis and the given function.](image)

4. Determine the area below the curve

\[
f(x) = \begin{cases} 
  x + 4, & \text{for } -4 \leq x < -1 \\
  -3(x + 1)^2 + 3, & \text{for } -1 \leq x < 0 \\
  x^{1/2}, & \text{for } 0 \leq x \leq 4
\end{cases}
\]

over the domain \(-4 \leq x \leq 4\).

**Solution:** We have that \(-3(x + 1)^2 + 3 = -3(x^2 + 2x + 1) + 3 = -3x^2 - 6x\). Since areas can be computed and added separately, we can
break the integral apart according to its pieces. We have

\begin{align*}
\text{Area} &= \int_{-4}^{-1} (x + 4) \, dx + \int_{-1}^{0} (-3x^2 - 6x) \, dx + \int_{0}^{4} x^{1/2} \, dx \\
&= \left[ \frac{x^2}{2} + 4x \right]_{-4}^{-1} + \left[ -x^3 - 3x^2 \right]_{-1}^{0} + \left[ \frac{2}{3}x^{3/2} \right]_{0}^{4} \\
&= \left[ \left( \frac{1}{2} - 4 \right) - \frac{16}{2} + 16 \right] + \left[ -1 + 3 \right] + \frac{2}{3}(8) \\
&= \cdots = \frac{71}{6}.
\end{align*}

5. Calculate the area of the closed region bound by the curves \( \sin(x) \) and \( \cos(x) \) over the interval \( 0 \leq x \leq 2\pi \).

**Solution:** We want the area given by:

\[
\begin{align*}
\text{Area} &= \int_{\pi/4}^{5\pi/4} (\sin(x) - \cos(x)) \, dx \\
&= \left[ -\cos(x) - \sin(x) \right]_{\pi/4}^{5\pi/4} \\
&= \left[ -\cos\left(\frac{5\pi}{4}\right) - \sin\left(\frac{5\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right) \right] \\
&= 4\left(\frac{1}{\sqrt{2}}\right) = 2\sqrt{2}.
\end{align*}
\]
6. Joe is driving at a speed of 18 m/s on a country road when a deer darts out in front of him. He begins immediately applying a constant deceleration of 4 m/s². Based on what you know about the relationship between position, velocity, and acceleration, determine whether Joe hits the deer or not, given that the deer is 50 meters away when he starts slowing down.

**Solution:** If we let \( p(t) \) denote the position of the vehicle at time \( t \), \( v(t) \) denote the car’s velocity, and \( a(t) \) denote its accelerate, we have the system of relationships

\[
\frac{d}{dt} p(t) = v(t) \\
\frac{d}{dt} v(t) = a(t).
\]

Our given information implies that \( a(t) = -4 \) and \( v(0) = 18 \). For simplicity, we will set \( p(0) = 0 \) (i.e. the initial point where we start is \( p = 0 \)). We have

\[
\frac{d}{dt} v(t) = -4.
\]

This implies that we need to find a function \( v(t) \) whose derivative is 4. We can easily determine that any function

\[
v(t) = -4t + C
\]

where \( C \) is an arbitrary constant will work. We can use the initial information \( v(0) = 18 \) to determine

\[
v(0) = 18 = -4(0) + C \implies C = 18.
\]

It follows that we have

\[
v(t) = -4t + 18
\]

We now use the first relationship to get

\[
\frac{d}{dt} p(t) = -4t + 18.
\]

In other words, we need to find a function \( p(t) \) which has the above expression as its derivative. We can easily determine that any function

\[
p(t) = -2t^2 + 18t + D
\]
where $D$ is an arbitrary constant will work. In order to solve for $D$, we use $p(0) = 0$ to get

$$p(0) = 0 = -2(0)^2 + 18(0) + D \implies D = 0.$$  

It follows that

$$p(t) = -2t^2 + 18t.$$  

The time when the vehicle comes to a complete stop is given by the time when the velocity is equal to zero. We have

$$v(t) = -4t + 18 = 0 \implies t = \frac{9}{2}.$$  

The position of the car at this point is given by

$$p\left(\frac{9}{2}\right) = -2\left(\frac{9}{2}\right)^2 + 18\left(\frac{9}{2}\right) = -\frac{81}{2} + 81 = \frac{81}{2} < 50.$$  

Since the distance Joe will travel before coming to a complete stop is less than fifty, it follows that he will stop in time to avoid hitting the deer.