Submission instructions: Clearly write your full name and ID number on the first page. To avoid marking and handling difficulties, please staple all submitted pages together and answer the questions in the order they appear on the assignment.

Academic integrity: Students are encouraged to collaborate on assignment problems but must write up their assignments independently. Copying is strictly forbidden!

Suggested problems:

Section 7.3, 1 − 12, 21, 22, 24 − 26
Section 6.6, 1 − 4, 10 − 13
Section 6.7, 1 − 6, 10
Section 7.4, 1 − 12
Section 7.5, 1 − 15

Problems for submission:

1. Determine the length of the arc

\[ f(x) = \frac{e^x + e^{-x}}{2} \]

from \( x = 0 \) to \( x = 1 \). (Hint: This can be done directly, but it can done faster using hyperbolic trigonometric identities.)

2. Consider a circular cone of length \( h \) and an open-end radius of \( r \). Show that the surface area of this cone is

\[ \pi r \sqrt{r^2 + h^2} . \]

3. Determine the surface area of the shape produced by wrapping the curve \( f(x) = 2\sqrt{x} \) from \( x = 0 \) to \( x = 1 \) around the \( x \)-axis.
4. Consider the shape bound by \( f(x) = x^n \) and \( g(x) = x^m \) where \( m > n > 0 \) between \( x = 0 \) and \( x = 1 \).

(a) Prove that the centroid of this shape can be given by
\[
x^* = \frac{(n + 1)(m + 1)}{(n + 2)(m + 2)}, \quad y^* = \frac{(n + 1)(m + 1)}{(2n + 1)(2m + 1)}.
\]

(b) Find the centroid of the shape bound by \( f(x) = \sqrt{x} \) and \( g(x) = x^2 \) between \( x = 0 \) and \( x = 1 \).

(c) What happens as \( n \to 0 \) and \( m \to \infty \)? To what shape does this region correspond?

5. Approximate the area under the curve \( f(x) = \sqrt{4 - x^2} \) between \( x = 0 \) and \( x = 2 \) using the rectangular (right end-point), trapezoidal, and Simpson’s rules using \( n = 4 \). Which method gives the best estimate of the actual solution \( \text{[Area]}=\pi \)?

6. How many intervals \( n \) must be chosen in order to guarantee an estimate accurately to \( E = 0.01 \) for the area under the curve of \( f(x) = \cos(x)e^{-x} \) between \( x = 0 \) and \( x = 4 \) using

(a) the trapezoidal rule, and

(b) Simpson’s rule.