MATH 133A, Fall 2015, Assignment 3
Due date: Tuesday, September 15 (in class)

Name (printed): ________________________________

SJSU Student ID Number: ________________________________

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.

<table>
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<th>Correctness</th>
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2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.

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3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

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| Bonus: | /3 |

Grader Initials: ________________________________
Q1: Find the general solution of the following differential equations:

(a) \[ y' = \frac{xy + y^2}{x^2} \]

(b) \[ x^2y' + 2xy - y^3 = 0, \quad x > 0 \]

(c) \[ y' = \frac{x + 3y}{x - y} \]

Q2: Solve the following initial value problems:

(a) \[
\begin{align*}
\frac{x}{(x^2 + y^2)^{3/2}} + \frac{y}{(x^2 + y^2)^{3/2}} + dy &= 0 \\
y(0) &= 1
\end{align*}
\]

(b) \[
\begin{align*}
2y^2 \, dx + (1 + 2xy) \, dy &= 0 \\
y(1) &= 1
\end{align*}
\]

Q3: Suppose that \( n \neq 0 \) and \( n \neq 1 \). Show that the substitution \( v = y^{1-n} \) transforms the Bernoulli equation

\[ y' + P(x)y = Q(x)y^n \]

into the first-order linear equation

\[ v' + (1-n)P(x)v = (1-n)Q(x) \].

Q4: On Assignment #1, we were introduced to the logistic model for population growth:

\[
\begin{align*}
\frac{dP}{dt} &= k \left(1 - \frac{P}{N}\right)P \\
P(0) &= P_0
\end{align*}
\]

(1)

where \( P(t) \) is the size of the population size at time \( t \), \( k > 0 \) is the growth rate, \( N > 0 \) is the carrying capacity, and \( P_0 \geq 0 \) is the initial population size. For simplicity, we will consider the parameter values \( k = 50, \quad N = 1000, \quad P_0 = 500 \).
(a) Show that the equation is *separable* and then use the associated method to determine the solution of the IVP. [*Hint:* You will need to use partial fractions decomposition!]

(b) Show that the equation is *Bernoulli* and then use the associated method to determine the solution of the IVP.

**BONUS** Suppose we add a harvesting term to the logistic equation in the form of a constant value $R > 0$. This could model, for example, a constant demand for lumber, fish, or animal pelts over time. This gives the *logistic plus harvesting* model

$$
\begin{align*}
\frac{dP}{dt} &= k \left( 1 - \frac{P}{N} \right) P - R \\
P(0) &= P_0
\end{align*}
$$

(a) Solve the initial value problem (2) with the parameter values $k = 50$, $N = 1000$, $R = 12500$. Keep the initial condition $P(0) = P_0$ symbolic. (*Hint:* Carefully factor before you integrate! The RHS will simplify.)

(b) Describe the long-term behavior of trajectories for the initial conditions $0 < P_0 < 500$, $P_0 = 500$, and $P_0 > 500$. What does the behavior in each region correspond to in the physical model? (*Hint:* Be careful to note any asymptotes in the solution!)