Math 32, Fall 2015, Term Test I (Sample)
Calculus III

Date: Thursday, September 24
Lecture Section: 001
Instructor: Matthew Johnston

Surname (Family Name): __________________________

Given Name: __________________________

SJSU Student ID Number: __________________________

Instructions
1. Fill out this cover page completely.
2. Answer questions in the space provided, using backs of pages for overflow and rough work.
3. Show all the work required to obtain your answers.
4. No calculators are permitted but you may consult a one page hand-written cheat sheet.

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1. Parametric Equations:

Consider the following parametric expression for $-2 \leq t \leq 2$:

\[
\begin{align*}
  x &= t^3 \\
  y &= 4t - t^3
\end{align*}
\]

(a) Sketch the curve.

(b) Rewrite as a single expression of $x$ and $y$.

(c) As with polar equations, we can determine the slope at a given point $t$ by using the formula:

\[
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}.
\]

Determine the slope $\frac{dy}{dx}$ when (i) $t = -2$, (ii) $t = 0$, and (iii) $t = 2$. 
2. Parametric Equations (con’t):

Consider the following parametric expression for $0 \leq t \leq \pi$:

\[
\begin{align*}
  x &= \cos(2t) \\
  y &= \cos(t)
\end{align*}
\]

(a) Sketch the curve.

(b) Rewrite as a single expression of $x$ and $y$. Reduce fully!

(\textbf{Hint: } \cos(2t) = 2\cos^2(t) - 1.)

(c) Use the parametric expression to determine all points $0 \leq t \leq 2\pi$ where the curve is vertical.

(\textbf{Hint: } Be careful of undefined points where we have “0/0”!)
3. Polar Equations:

Consider the following polar expression:

\[ r = \cos(3\theta), \ 0 \leq \theta \leq \pi. \]

(a) Sketch the curve.

(b) Determine the slope of the curve when \( \theta = \pi/3 \).

(c) The curve passes through \((0, 0)\) three times. Determine the slope of the curve as it passes through each time.
4. Polar Equations (con’t):

Consider the following polar expression:

\[ r = 1 + \sin(2\theta), \quad 0 \leq \theta \leq 2\pi. \]

(a) Sketch the curve.

(b) Determine the slope when \( \theta = \frac{\pi}{4} \).

(c) Determine the slope when \( \theta = \frac{3\pi}{4} \). Explain what happens at this point.
   \textbf{(Hint:} Carefully consider the sketch in part (a)!)
5. Dot and Cross Product:

Consider \( \mathbf{a} = \langle -1, 6, -5 \rangle \) and \( \mathbf{b} = \langle 0, 2, -1 \rangle \).

(a) Determine the angle between \( \mathbf{a} \) and \( \mathbf{b} \). You do not need to simplify. Is the angle acute or obtuse?

(b) Determine a vector which is perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \).

(c) Determine all values of \( t \) for which is \( \mathbf{c} = \langle t^2, 1, t \rangle \) is perpendicular to \( \mathbf{a} \).
6. Lines and Planes:

Consider the points \( A = (1, -3, 2) \), \( B = (-5, 3, 2) \), and \( C = (2, 0, -2) \).

(a) Determine the parametric form of the line through the points \( A \) and \( B \).

(b) Determine the angle \( \angle ABC \). You do not need to simplify your answer. Is the angle acute or obtuse?

(c) Determine the equation of the plane containing \( A \), \( B \), and \( C \).
7. Lines and Planes (con’t):

Consider the points $A = (2, 3, 0)$, $B = (0, 1, 1)$, and $C = (-2, k, 2)$.

(a) Determine the value of $k$ for which $A$, $B$, and $C$ lie on the same line.

(b) For all values of $k$ other than that derived in part (a), find the equation of the plane through $A$, $B$, and $C$. Does anything interesting happen when $k$ takes the value in part (a)? Why does this happen?

(Note: Keep $k$ undetermined.)
8. Lines and Planes (con’t):

(a) Determine the projection of \( \mathbf{a} = (-1, 5, 0) \) onto the line with symmetric form

\[
\frac{x - 1}{2} = \frac{y + 5}{4} = \frac{z - 2}{3}.
\]

(b) Consider the following planes:

Plane 1: \( x + 2y + 5z = 0 \)

Plane 2: \( -2y + z = 0. \)

Determine the parametric form of the line of intersection between the two planes.
9. **Quadric Surfaces:**

Consider the following equation:

\[ x^2 + 4y^2 - z = 4. \]

(a) Sketch the traces \( x = k, y = k, \) and \( z = k \) in their respective planes. Classify each trace as either a parabola, circle, ellipse, or hyperbola.

(b) Sketch the surface in the \((x, y, z)\)-plane.
10. Quadric Surfaces (con’t):

Consider the following equation:

\[ x^2 - y^2 + z = 0. \]

(a) Sketch the traces \(x = k, y = k,\) and \(z = k\) in their respective planes. Classify each trace as either a parabola, circle, ellipse, or hyperbola.

(b) Sketch the surface in the \((x, y, z)\)-plane.
THIS PAGE IS FOR ROUGH WORK