Instructions

1. Fill out this cover page completely and affix it to the front of your submitted assignment.

2. Staple your assignment together and answer the questions in the order they appear on the assignment sheet.

3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. Copying is strictly forbidden!

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Grader Initials: [ ]
Systems of First-Order Equations

Q1: Transform the following differential equations into a system of first order differential equations (all derivatives are with respect to $x$):

(a) $x^2 y'' + xy' + (x^2 - 0.25)y = 0$
(b) $y^{(4)} - y = 0$

Q2: Sketch the vector field in the $(x, y)$-plane for the following systems of first-order differential equations (all derivatives are with respect to $t$):

(a) \[
\begin{cases}
x' = x - y \\
y' = 2x - 5y
\end{cases}
\]
(b) \[
\begin{cases}
x' = -x + 3y \\
y' = x - 3y
\end{cases}
\]

Q3: Find the general solution of the following systems of differential equations (all derivatives are with respect to $t$):

(a) \[
\begin{cases}
x' = x - 6y \\
y' = x - 4y
\end{cases}
\]
(b) \[
\begin{cases}
x' = 4x - 2y \\
y' = 6x - 3y
\end{cases}
\]

Q4: Verify that

\[
x(t) = \begin{bmatrix} e^{-t(-t^2 + 6t)} \\ e^{-t(-2t + 2)} \\ e^{-t(t^2 - 8t)} \end{bmatrix}
\]

is a solution of the following initial value problem:

\[
\begin{cases}
x' = -3x + 3y - 2z, & x(0) = 0 \\
y' = x - 2y + z, & y(0) = 2 \\
z' = 3x - 4y + 2z, & z(0) = 0.
\end{cases}
\]

BONUS: Consider the system in Q3(a). Use the transformation

\[
\begin{cases}
u = x - 2y \\
v = -x + 3y
\end{cases}
\]
to rewrite the system in $x$ and $y$ as a system in $u$ and $v$. Solve this system and then use it to determine the solution for $x$ and $y$. (Hint: Note that the variable transformation can be written in matrix form as $u = Px$ where $u = (u, v), x = (x, y)$, and $P$ is the coefficient matrix. To transform back to $x$ and $y$, therefore, we need $x = P^{-1}u$.)