Math 32, Fall 2015, Term Test I
Calculus III

Date: Thursday, September 24
Time: 10:30-11:45 a.m.
Lecture Section: 001
Instructor: Matthew Johnston

Surname (Family Name): ____________________________
Given Name: ____________________________
SJSU Student ID Number: ____________________________

Instructions

1. Fill out this cover page completely.

2. Answer questions in the space provided, using scratch paper for rough work.

3. Show all the work required to obtain your answers.

4. No calculators are permitted but you may consult a one page hand-written cheat sheet.

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1. **True/False:**

(a) The curve $\mathbf{r}(t) = \langle t, t^2 - 1 \rangle$ is a parametrization of $y = x^2 - 1$. [True / False]

(b) Suppose that a curve has polar form $r = f(\theta)$. Then the corresponding parametrization over $\theta$ is $\mathbf{r}(\theta) = \langle f(\theta) \cos(\theta), f(\theta) \sin(\theta) \rangle$. [True / False]

(c) In three-dimensions, it is possible for three vectors to be mutually perpendicular (i.e. $\mathbf{v}_1 \perp \mathbf{v}_2$, $\mathbf{v}_1 \perp \mathbf{v}_3$, and $\mathbf{v}_2 \perp \mathbf{v}_3$). [True / False]

(d) The projection of the vector $\mathbf{a}$ onto $\mathbf{b}$ has magnitude zero if and only if $\mathbf{a} \perp \mathbf{b}$. [True / False]

(e) An equivalent characterization of the magnitude of a vector $\mathbf{a}$ is given by $|\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$. [True / False]

(f) Consider a plane given by $ax + by + cz = d$. Then the vector $\mathbf{n} = \langle a, b, c \rangle$ is parallel to the plane. [True / False]

(g) Of the three traces of $x^2 - y^2 - z^2 = 0$, two are circles and one is a hyperbola. [True / False]
2. Parametric Equations:

Consider the parametric curve \( r(t) = (x(t), y(t)) \) where:

\[
\begin{align*}
  x(t) &= 2t - t^2 \\
  y(t) &= 4t - t^2
\end{align*}
\]

(a) Sketch the functions \( x(t) \) and \( y(t) \) (separately) over the range \( 0 \leq t \leq 4 \).

\textbf{Hint:} Consider the roots of the parabolas!

(b) Sketch \( r(t) \) in the \((x,y)\)-plane over the range \( 0 \leq t \leq 4 \). Indicate the direction of
the curve for increasing \( t \).

(c) As with polar equations, we can determine the slope at a given point \( t \) by using
the formula:

\[
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}.
\]

Determine the point \((x, y)\) where the tangent to \( r(t) = (x(t), y(t)) \) is vertical and
the point \((x, y)\) where it is horizontal.
3. Polar Equations:

Consider the following polar expression:

\[ r = 1 + \cos(\theta), \quad 0 \leq \theta \leq 2\pi. \]

(a) Sketch the curve in the \((x, y)\)-plane.

(b) Rewrite the expression in Cartesian coordinates. (Note: You can leave it in implicit form.)

(c) Determine the slope when \( \theta = \frac{\pi}{4} \).

(d) Determine the slope when \( \theta = \pi \). Explain what happens at this point.

\textbf{Hint:} Carefully consider the sketch in part (a)!
4. Dot and Cross Product:

Consider the vectors \(\mathbf{a} = \langle -2, 1, -1 \rangle\) and \(\mathbf{b} = \langle 3, 5, 0 \rangle\).

(a) Determine the projection of \(\mathbf{b}\) onto \(\mathbf{a}\), \(\text{proj}_\mathbf{a}(\mathbf{b})\).

(b) Determine a vector which is perpendicular to both \(\mathbf{a}\) and \(\mathbf{b}\).

(c) Determine values of \(t\) for which \(\mathbf{c} = \langle t, 1, t^2 \rangle\) is perpendicular to \(\mathbf{a} - \mathbf{b}\).

(d) Determine values of \(k\) for which \(\mathbf{c} = \langle k + 3, k, 1 \rangle\) is collinear to \(3\mathbf{a}\).
5. Lines and Planes:

Consider the points \( A = (3, 0, 1) \), \( B = (0, -1, 2) \), and \( C = (3, 3, -5) \).

(a) Determine the angle \( \angle CAB \). [2]

(b) Determine the equation of the plane containing \( A \), \( B \), and \( C \). [2]

(c) Determine if the vector \( \mathbf{v} = \langle 3, 0, 1 \rangle \) lies parallel to the plane found in part (b). [2]
6. **Quadric Surfaces:**

Consider the following equation:

\[ x^2 - y^2 + z = 0. \]

(a) Sketch at least two of the traces \( x = k, \ y = k, \) or \( z = k \) in their respective planes. Classify the traces you choose as either parabolas, circles, ellipses, or hyperbolas.

(b) Show that the following parametrization lies on the surface for \( t \in \mathbb{R}, \ r \in \mathbb{R}: \)

\[
\begin{align*}
  x &= r \left( \frac{e^t + e^{-t}}{2} \right) \\
  y &= r \left( \frac{e^t - e^{-t}}{2} \right) \\
  z &= -r^2
\end{align*}
\]
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