MATH 133A, Fall 2015, Assignment 8
Due date: Thursday, November 5 (in class)

Name (printed): ________________________________
SJSU Student ID Number: ________________________

Instructions

1. Fill out this cover page completely and affix it to the front of your submitted assignment.

2. Staple your assignment together and answer the questions in the order they appear on the assignment sheet.

3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. Copying is strictly forbidden!

<table>
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Grader Initials: ____________________________
Nonhomogeneous Systems and Second-Order DEs

Q1: Find the general solution of the following nonhomogeneous system of differential equations (all derivatives are with respect to $t$):

$$\begin{cases} x' = 4x - 2y + t^3 \\ y' = 8x - 4y - t^2 \end{cases}$$

Q2: Solve the following second-order differential equations (all derivatives with respect to $x$):

(a) $y'' + y' - 6y = 0$, \quad $y(0) = 1$, $y'(0) = 2$
(b) $9y'' + 6y' + y = 0$, \quad $y(0) = 3$, $y'(0) = -2$
(c) $y'' - 4y' + 13y = 0$, \quad $y(0) = 5$, $y'(0) = 1$

Q3: Verify the following second-order differential equations have the following particular solutions, and then solve the given initial value problem:

(a) $2y'' + 5y' - 3y = 4e^x$, \quad $y_p(x) = e^x$
\quad $y(0) = 0$, $y'(0) = -3$
(b) $y'' + 2y' + 5y = 16xe^x$, \quad $y_p(x) = (2x - 1)e^x$
\quad $y(0) = -1$, $y'(0) = 1$

Q4: Consider a reversible chemical reaction of the form $X \rightleftharpoons Y$ occurring in a reaction chamber subject to continuous inflow of $X$ and outflow of both $X$ and $Y$. Suppose the system is governed by the following system of ordinary differential equations:

$$\begin{cases} x' = 3 - 2x + y, \quad x(0) = 0 \\ y' = x - 2y, \quad y(0) = 1 \end{cases}$$

where $x(t)$ and $y(t)$ are the concentrations of $X$ and $Y$, respectively, at time $t$.

(a) Construct a vector field diagram for (1) in the region $x \geq 0$, $y \geq 0$. [Note: The nullclines are lines, but they do not all go through $(0,0)$!]

(b) Solve the system (1) for the concentrations $x(t)$ and $y(t)$. 
(c) Describe the long-term behavior of the solution obtained in part (b). Does this make sense in terms of the vector field obtained in part (a)? Explain.

**BONUS:** Use the following substitution to rewrite the system (1) in Q4 as a system in the variables $u$ and $v$:

$$
\begin{align*}
  u &= \frac{1}{2} (-x + y + 1) \\
  v &= \frac{1}{2} (x + y - 3).
\end{align*}
$$

Solve the resulting system for $u(t)$ and $v(t)$, and then use the substitution to return to the variables $x(t)$ and $y(t)$. **Hint:** Notice that the substitution can be written as

$$
u = Px + b$$

where

$$
u = \begin{bmatrix} u \\ v \end{bmatrix}, P = \frac{1}{2} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, x = \begin{bmatrix} x \\ y \end{bmatrix}, \text{ and } b = \frac{1}{2} \begin{bmatrix} 1 \\ -3 \end{bmatrix}.$$

This implies that

$$
x = P^{-1} (u - b).$$