Name (printed): __________________________

SJSU Student ID Number: __________________________

**Instructions**

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.

   ![Correctness Table]
   
   Correctness /15

2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.

   ![Completeness Table]
   
   Completeness /5

3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

   ![Total and Bonus Tables]
   
   Total: /20
   Bonus: /3

**Grader Initials:**
Laplace Transforms

Q1: Use the definition of the Laplace Transform to evaluate the following:

(a) \( \mathcal{L}\{xe^{ax}\}, \ a \in \mathbb{R} \)  \hspace{1cm} (b) \( \mathcal{L}\{\cos(bx)\}, \ b \in \mathbb{R} \)

Q2: Determine the Laplace transform of the following:

(a) \( f(x) = e^{-x} - x^4 + \cos(2x) \)
(b) \( f(x) = e^{3x} \ (x + \sin(x)) \)

(Note: You do not have to use the definition!)

Q3: Determine the inverse Laplace transform of the following:

(a) \( F(s) = \frac{s^2 + 1}{s^3 - 2s^2 + s} \)
(b) \( F(s) = \frac{2s + 1}{s^2 - 2s + 2} \)

Q4: Use Laplace transforms to solve the following initial value problems:

(a) \( y'' - 4y' + 4y = 0; \ y(0) = 1, \ y'(0) = 1 \)
(b) \( y'' + 2y' + y = 4e^{-x}; \ y(0) = 2, \ y'(0) = -1 \)

BONUS: One notable exception to our list of Laplace transform identities has been

\[ \mathcal{L}\{x^n f(x)\} \]

That is to say, we have no general identity for the Laplace transform of a standard function multiplied by a power of \( x \).

Suppose that \( \mathcal{L}\{f(x)\} = F(s) \). Use the definition of the Laplace transform to show that \( \mathcal{L}\{xf(x)\} = -F'(s) \). Use this to evaluate

\[ \mathcal{L}^{-1}\left\{ \frac{4s}{(s^2 + 4)^2} \right\} \]