MATH 133A, Fall 2015, Assignment 11
Due date: Tuesday, December 8 (in class)

Name (printed): ____________________________

SJSU Student ID Number: ____________________________

Instructions

1. Fill out this cover page completely and affix it to the front of your submitted assignment.

2. Staple your assignment together and answer the questions in the order they appear on the assignment sheet.

3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. Copying is strictly forbidden!

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Grader Initials: ____________________________
Heaviside and Dirac Delta Functions

Q1: For the following piecewise-defined functions \( f(x) \), sketch the functions and then rewrite it as a single expression using Heaviside functions:

\[
\begin{align*}
(a) \quad f(x) &= \begin{cases} 
1, & 0 \leq x < 1 \\
2 - x, & 1 \leq x < 2 \\
0, & x \geq 2.
\end{cases} \\
(b) \quad f(x) &= \begin{cases} 
2x - x^2, & 0 \leq x < 2 \\
x^2 - 6x + 8, & 2 \leq x < 4 \\
0, & x \geq 4.
\end{cases} \\
(c) \quad f(x) &= n, \quad n \leq x < n + 1, \; n \in \{0, 1, 2, \ldots\}.
\end{align*}
\]

Q2: Determine the Laplace transform of the following:

\[
\begin{align*}
(a) \quad f(x) &= u_2(x) - u_4(x) \\
(b) \quad f(x) &= 1 + (2 - x)u_1(x) + (x - 3)^2u_2(x) \\
(c) \quad f(x) &= \delta(x - 1) - 2\delta(x - 2) + \delta(x - 3)
\end{align*}
\]

(Note: You do not have to use the definition!)

Q3: Determine the inverse Laplace transform of the following:

\[
\begin{align*}
(a) \quad F(s) &= \frac{12se^{-2s}}{3s^2 - 2s - 1} \\
(b) \quad F(s) &= \frac{e^{-\pi s} + e^{-2\pi s}}{s^3 - s^2 + 4s - 4} \quad \textbf{[Hint: Note that } s = 1 \text{ is a root!]} \\
\end{align*}
\]

Q4: Use Laplace transforms to solve the following initial value problem:

\[
y'' + 7y' + 10y = u_1(t) - u_2(t); \quad y(0) = 3, \; y'(0) = 0
\]

Q5: Suppose an undamped pendulum weighting 1kg and with a restoring force constant of 9N/m is extended 1m to the right and then released from rest. Suppose furthermore that there is a right-ward impulse applied to the pendulum at \( t = \frac{2}{3}\pi \) s. This gives rise to the model.

\[
x'' + 9x = \delta \left( t - \frac{2}{3}\pi \right); \quad x(0) = 1, \; x'(0) = 0. \quad (1)
\]

(a) Solve the initial value problem (1).

(a) Determine the amplitude of harmonic motion in the interval \( 0 \leq t < \frac{2}{3}\pi \) and in the interval \( t \geq \frac{2}{3}\pi \). How do they compare? Does this make sense? Why or why not?