

# Math 133A, Spring 2016, Term Test I

## Ordinary Differential Equations

**Date:** Wednesday, March 2

**Time:** 12:00-1:15 p.m.

**Lecture Section:** 001

**Instructor:** Matthew D. Johnston

Last Name: \_\_\_\_\_

First Name: \_\_\_\_\_

SJSU Student ID Number: \_\_\_\_\_

### Instructions

1. Fill out this cover page completely.
2. Answer questions in the space provided, using scratch paper for rough work.
3. Show all the work required to obtain your answers.
4. No calculators are permitted but you may consult a one page hand-written cheat sheet.

FOR EXAMINERS' USE ONLY	
Page	Mark
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5	/5
6	/6
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1. Short Answer:

- [2] (a) Give an example of an **exact differential equation** (in standard form, please!). Use a result from class to verify that the selected equation is, in fact, exact.

- [1] (b) Determine the **integrating factor** for the following first-order linear differential equation (all derivatives with respect to  $x$ ):

$$y' + \left( \cos(x) - \frac{3}{x} \right) \cdot y = 3x^2.$$

2. True/False:

- [1] (a) The solution  $y(x)$  to the initial value problem  $\{y' = f(x, y), y(x_0) = y_0\}$  corresponds to the curve through the point  $(x_0, y_0)$  in the slope field diagram. [True / False]
- [1] (b) A Bernoulli differential equation may always be transformed into a separable differential equation by the substitution  $v = y/x$ . [True / False]
- [1] (c) The initial value problem  $\{y' = x + \frac{1}{y}, y(0) = 1\}$  has a unique solution through the point  $(x_0, y_0) = (0, 1)$ . [True / False]

3. Slope Fields and Solutions:

Consider the following initial value problem:

$$\begin{cases} \frac{dy}{dx} = x - 2y \\ y(0) = -\frac{1}{4} \end{cases} \quad (1)$$

- [3] (a) Sketch the slope field of (1) in the  $(x, y)$ -plane and then overlay a few solutions. [Hint: Setting  $y' = C$  gives  $y = \frac{1}{2}x - \frac{C}{2}$ .]

- [2] (b) Verify that  $y(x) = \frac{1}{2}x - \frac{1}{4}$  is a solution of (1). [Hint: Remember to check the initial condition!]

- [1] (c) Is the solution verified in part (b) the unique solution to through  $(0, -\frac{1}{4})$ . Justify your answer.

4. First-Order Differential Equations:

Solve the following first-order differential equations:

$$[3] \quad \text{(a)} \quad \frac{dy}{dx} = 2 \cdot \left(\frac{y}{x}\right) - \frac{x}{y}, \quad y > 0$$

$$[4] \quad \text{(b)} \quad \begin{cases} \left(\frac{2x}{y}\right) dx + \left(2y - \frac{x^2}{y^2}\right) dy = 0, \\ y(0) = 3 \end{cases} \quad \text{[Note: Don't forget the initial condition!]}$$

5. Applications: (Inflow/Outflow)

Consider a mixing tank which initially contains 16 gallons of fresh water. Suppose there is an inflow pipe which pumps in a 0.25 lb/gallon brine (salt/water) mixture at a rate of 8 gallons per minute, and there is an outflow pipe which removes the mixture from the tank at a rate of 8 gallons per minute.

[1] (a) Use the given information to derive an initial value problem which models the amount of salt in the tank over time. (**Note:** Do not forget about the initial condition!)

[3] (b) Find the particular solution of the initial value problem derived in part (a).

[1] (c) What is the limiting amount of salt in the tank as  $t \rightarrow \infty$ ? Interpret this value in terms of the inflow and outflow of the system.

6. Applications: (Terminal Velocity)

Consider a skydiver in freefall. Suppose that the rate at which the diver's velocity  $v(t)$  changes is proportional to the difference between the current velocity and the terminal freefall velocity (roughly 2 miles per minute). Specifically, suppose we have

$$\begin{cases} \frac{dv}{dt} = -2 - v \\ v(0) = 0 \end{cases} \quad (2)$$

where we assume a negative value of  $v(t)$  corresponds to a positive velocity in the downward direction.

- [3]      (a) Solve the initial value problem (2) for the velocity  $v(t)$ .
- [2]      (b) Keeping in mind that velocity is the derivative of position (i.e.  $v(t) = \frac{dx}{dt}$ ), solve for the position  $x(t)$  with the initial condition  $x(0) = 2$  (i.e. an initial jump from 2 miles above the Earth).
- [1]      (c) Has the skydiver reached the ground after 2 minutes (i.e.  $t = 2$ )? Explain your reasoning. (**Note:** The ground corresponds to  $x = 0$ !)

THIS PAGE IS FOR ROUGH WORK